ON RATIO ESTIMATE IN COST OF PRODUCTION STUDIES

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Summary

In the study of cost of production of a commodity, the cost per unit is obtained by taking the ratio of total cost of all units to the number of units produced. Numerator in this is generally a sum of product of two random variables. In the present study formulae have boen obtained for relative bias and variance of estjmate of (R_n) by considering the variable occuring in the numerator as (i) a single variate (ii) product of two independent variates and (iii) product of two dependent variates. For the latter two cases expressions for variances have been obtained using approximate and exact formulae given by Goodman.

Introduction

In cost of production studies, the formula for the estimates of the components of costs per unit generally contains numerator as a sum of products of two random variables. It is assumed therein that the results obtained by treating the numerator as a single variable only, would not differ from those obtained by considering it as a product of two variables. This is not however statistically studied.

Kendall and Stuart [3] have given an approximate formula for the variance of the product of two random variables. This approximate formula is satisfactory only if the coefficients of variation of the two random variables are both relatively small. An exact formula given by Goodman [1] does not depend on any such assumption concerning the magnitude of the coefficients of variation. Both these methods can be used for estimating bias and variances in the cost of production studies, separately when two random variables are (*i*) dependent or (*ii*) independent.

The main object of the study is to find the ratio estimates treating these factors in the numerator as separate random variables which vary over farms. Further estimates for variances of these ratios are modified in the light of the exact formula, given by Goodman [1] The relative bias and variances of these estimates are discussed. To illustrate the use of these expressions for relative bias and variance, the data from a project "Cost of production of poultry and eggs" Delhi (Marutiram *et al.* [4]) were analysed. Cost on mash, which is a composite feed, per bird was obtained together with relative bias and C.V. for the estimate.

2. Estimate of Cost per unit and the Bias :

Let

N = total number of commercial poultry farms.

n = number of commercial poulty farms selected for the study.

z_i = total feed cost on the ith farm.

P_i = Rate/Kg. of feed fed to poultry birds at ith farm.

q_i = Quantity of feed in Kg. fed to poultry birds on the ith farm.

x_i = total number of adult birds on ith farm.

$$R_N = \frac{\frac{1}{N}\sum_{i=1}^{N} z_i}{\frac{1}{N}\sum_{i=l}^{N} x_i} = \frac{\text{Total feed cost}}{\text{Total number of birds}}$$

$$= \text{Cost of maintenanc per bird}$$

$$R_N = \frac{\frac{1}{N}\sum_{i=l}^{N} P_i q_i}{\frac{1}{N}\sum_{i=l}^{N} x_i}$$

$$A = \left[1 + \left(\frac{N-1}{N}\right) \rho_{pq} \; C_p C_q \right]$$

$$S_{ij}=\frac{1}{(N-1)}\sum_{i=l}^{N}\left(p_i-\overline{P_N}\right)^i\left(q_i-\overline{q_N}\right);$$

$$C_p=\frac{\sum(p_i-\bar{P}_N)^2}{(N-1)\,\overline{P}_N}$$

The bias and variance of the estimate of cost per unit will be discussed for the following three cases arising by considering the variable in the numerator as

(*i*) a single variate

(*ii*) product of two independent variates

(*iii*) product of two dependent variates.

Symbolically, these cases can be written as

$$(i)\quad R_n=\frac{\frac{1}{n}\sum_{i=l}^{n} z_i}{\frac{1}{n}\sum_{i=1}^{n} x_i}=\frac{\bar{z}_n}{\bar{x}_n}$$ where $z_i=(pq)_i$ and is

considered as a single variate.

$$(ii)\quad R_n=\frac{\frac{1}{n}\sum_{i=1}^{n} p_iq_i}{\frac{1}{n}\sum_{i=1}^{n} x_i}=\frac{\bar{y}_n}{\bar{x}_n}$$

where p_i and q_i are independent variates

q_i and x_i are dependent variates.

$$(iii)\quad R_n\frac{\frac{1}{n}\sum_{i=}^{n} p_iq_i}{\frac{1}{n}\sum_{i=}^{n} x_i}=\frac{\bar{y}_n}{\bar{x}_n}$$

where p_i and q_i are independent variates.

q_i and x_i are dependent variates.

The relative biases of the above three cases obtained by usual method [5] are as follows :

$$(i)\quad \frac{N-n}{nN}[C_x^2-\rho_{xz}\,C_xC_z]$$

(ii) $\frac{N-n}{nN}[C_x^2-(\rho_{xq}C_xC_q+\rho_{xp}C_xC_p)]$

(iii) $\dfrac{\frac{N-n}{nN}[C_x^2-(\rho_{xq}C_q+\rho_{xp}C_xC_p)]}{1+\left(\frac{N-1}{N}\right)\rho_{pq}C_pC_q}$

3. VARIANCES OF THE ESTIMATES

The variance of first estimate is obtained using general method for standard errors of functions of random variables (Kendall and Stuart [3]. The population variances of the II & III estimates are obtained by using the approximate and exact formulae for obtaining the variances of the product of two independent and dependent variables given by Goodman [1].

The variances of all the three estimates are as follows

$$(i)\ V_1(R_n)=\left(\frac{N-n}{nN}\right)\left(\frac{\bar{z}_N}{\bar{x}_N}\right)^2\ [C_z^2+C_x^2-2\rho_{xz}C_xC_z]$$

(ii) A : using approximate formula

$$V_{2app.}(R_n)=\left(\frac{N-n}{nN}\right)\left(\frac{\bar{P}_N\bar{Q}_N}{X_N}\right)[C_q^2+C_p^2+C_x^2-2\ (\rho_{xq}C_xC_q+\rho_{xp}C_xC_p)$$

(ii) B : using approximate formula

$$V_{2exact}(R_n)=\left(\frac{N-n}{nN}\right)\left(\frac{\bar{P}_N\bar{Q}_N}{\bar{X}_N}\right)^2[C_p^2+C_q^2+C_x^2 - (\rho_{xq}C_xC_q+\rho_{xp}C_xC_p)+\left(\frac{N-1}{N}\right)(C_p^2C_q^2)$$

$$=V_{2app}(R_n)+\left(\frac{N-n}{nN}\right)\left(\frac{N-1}{N}\right)\frac{\bar{P}_N\bar{Q}_N}{\bar{X}_N}\ C_p^2C_q^2$$

(iii) A : using approximate formula

$$V_{3app.}(R_n)=\left(\frac{N-n}{nN}\right)\left(\frac{\bar{P}_N\bar{Q}_N}{\bar{X}_N}\right)^2(A)^2\left[\frac{C_p^2+C_p^2+\rho_{pq}C_pC_q}{A^2}+C_x^2-\frac{2(\rho_{xq}C_xC_q+\rho_{xp}C_xC_p)}{A}\right]$$

(*iii*) *B* : *using exact formula*

$$V_{3exact}(R_n)=\left(\frac{N-n}{nN}\right)\left(\frac{\bar{P}_N\bar{Q}_N}{\bar{X}_N}\right)^2 (A)^2 \left[\left\{C_q^2+C_p^2+2\left(\frac{S_{11}}{\bar{P}_N\bar{Q}_N}\right.\right.\right.$$

$$\left.+\frac{S_{12}}{\bar{P}_N\bar{Q}_N^2}+\frac{S_{21}}{\bar{P}_N^2\bar{Q}_N}\right)+\frac{S_{22}-\frac{N-1}{N}S_{11}^2}{\bar{p}_N^2\bar{q}_N^2}\Big\}/A^2+C_x^2$$

$$-\frac{2(\rho_{xq}C_xC_q\rho_{xp}C_xC_p)}{A}$$

The estimates of the population variance are obtained by replacing $\bar{Z}_N$, $\bar{X}_N$, Cz, C_x, ρ_{xz}, $\bar{P}, _N\bar{Q}_N$, C_q, C_p, ρ_{xq}, ρ_{xp}, ρ_{pq} S_{ji} and A by $\bar{z}_n$ $\bar{x}_n$, c_z, c_x, $\hat{\rho}_{xz}$,$\bar{p}_n$, $\bar{q}_n$, c_q, c_p, $\hat{\rho}_{xq}$, $\hat{\rho}_{xp}$, $\hat{\rho}_{pq}$, s_{ij} and $\hat{A}$ respectively, in the expression for population variance.

4. Relative bias and C.V. :

Mash which is a composite poultry feed formed about 99 percent of the total feed fed to a bird, other feeds fed being greens and marble chips. Readymade mash is obtained by different commercial poultry owners in different quantities. It's price differed from farm to farm. Average price per kg. of readymade mash fluctuated between 56 paise to 69 paise during the year 1970-71 in Delhi with mean 62 paise and *S.E.* 04. Estimates of relative bias and *C.V.* in estimating the average cost on mash per bird by different methods are given in the Table.

TABLE

Relative bias C.V. (%) by different methods.

Sl. No.	*Methods*	*Relative bias*	*C.V.*
1.	I	0.254	3.89
2.	II (approximate)	0.144	1.91
3,	II (exact)	0.144	2.38
4.	III (approximate)	0.141	3.45
5.	III (exact)	0.141	3.19

The table shows that by considering the variable in the numerator as a product of two independent variqtes the relative bias is decreased by 0.11% and it is further decreased by 0.003% by considering the variables in the numerator as a product of two dependent variates. The coefficient of variation are also decreased by 1.98% in II (approximate), 1.51% in II (exact), 0.44% in III (approximate) and 0.70% in III (exact) method respectively. The C.V. in II (exact) is 0.47% higher than that by II (app.) method. The C.V. in case of III (exact) is 0.26% less than the C.V. of III (approximate). This is due to the fact that the value of extra term figuring in the formula is negative. C.V. is least in II (app.) method.

5. Conclusion

Since in cost of production study consistent and unbiased estimates of cost are required, the above refinement gives more correct value of the relative bias and C.V. in the ratio estimate which may be less, equal to or more than the value obtained by considering the variable in the numerator as a single variate. Approximate and exact estimates arising out of Goodman formula do not change the relative bias. However, C.V. by II (approximate) method, is aways found to be less than by II (exact) method. This is because of the extra term

$$\left(\frac{N-n}{N}\right)\left(\frac{N-1}{N}\right), \left(\frac{P_N Q_N}{x_N}\right)^2 C_p^2 C_q^2$$

figuring in case of II (exact) method. The C.V. obtained by III (approximate) method may be less or more than that obtained by III (exact) method depending upon the sign of the value of the term

$$2\left(\frac{S_{12}}{P_N Q_N^2}+\frac{S_{21}}{P_N^2 Q_N}\right)+\left(\frac{S_{22}-\frac{N-1}{N}\, S_{11}^2}{P_N^2\, Q_N^2}\right)$$

figuring in II (exact method).

Thus, in cost of production study in will be preferable to use these formulae when there are reasons to presume that variable in the numerator is a product of two variates and by applying these formulae, one will be able to get more correct estimates of relative bias and C.V. than there estimated by considering the variable in the numerator as a single variate.

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